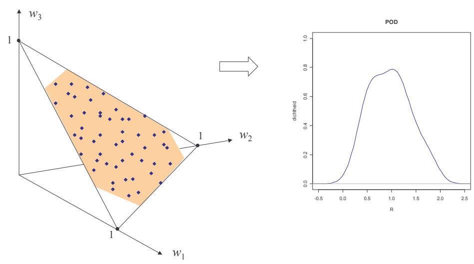
Portfolio Opportunity Distributions



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Master thesis

Portfolio Opportunity Distributions (PODs):

- Technique for Performance Evaluation.
- Rather unknown at the moment.
- Quite different from traditional methods:
    - Benchmarks;
    - Peer groups.
- Makes use of computer simulations.
- R. Surz e.a. (1996).

Contents

- Weaknesses of benchmarks and peer groups
- PODs: how it works globally
- Technical part: 3 algorithms
- Examples
- Conclusions

Benchmark

An index representing an underlying financial market.

- Market index.
  (FTSE index, MSCI World, S&P 500, …)
- Customized index.

Excess return / Value added

$R_1^P, \ldots, R_n^P$: portfolio returns

$R_1^M, \ldots, R_n^M$: benchmark returns

$$VA_i := R_i^P - R_i^M \quad \text{(arithmetic)}$$

$$:= \frac{1 + R_i^P}{1 + R_i^M} - 1 \quad \text{(geometric)}$$

Is performance good?

Statistical test: $H_0: \quad \mathbb{E}VA \leq 0$

$H_1: \quad \mathbb{E}VA > 0$

Reject the zero hypothesis at great values of

$$T_n := \sqrt{n}\, \frac{\overline{VA}_n}{\sigma_{VA,n}}$$

- VA_i normally distributed: t-test.
- If not: approximate by CLT, or: use a sign test.

Problems with benchmarks

- Statistical significnce must be obtained over time:

  1 observation per month $\Rightarrow$
  waiting more than 8 years for 100 observations!

- Specific mandate rules are not reflected by the benchmark:

Problems with benchmarks

Mandate A:	**Mandate B**:
European stocks	European stocks
Tracking Error $\leq$ 10%	Tracking Error $\leq$ 15%
Max. exposure 15%	Max. exposure 20%

Benchmark: MSCI Europa

Both managers choose the same portfolio $\Rightarrow$
both are assigned an equal performance score.
This is quite unfair !

Peer group

Group of managers with a similar mandate over the same time period.

R^P : portfolio returns

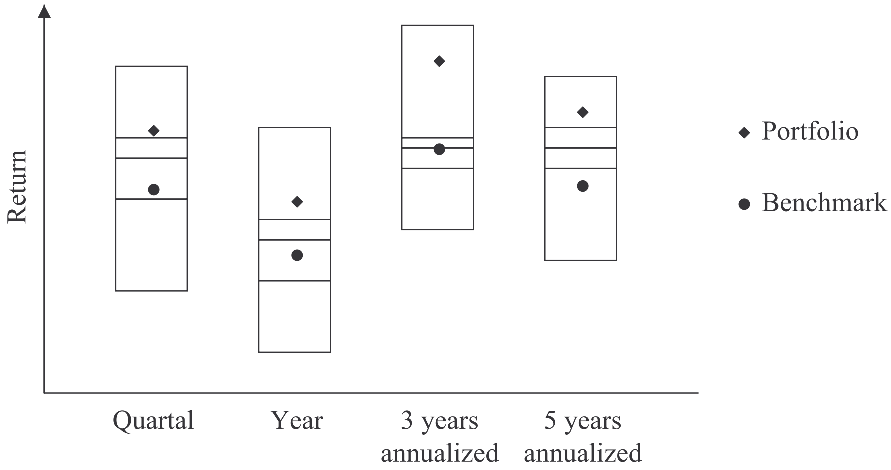
$R^{P_1},\dots,R^{P_m}$: peer group returns

Idea: compare R^P with $R^{P_1},\dots,R^{P_m}$.

Is performance good?

$$\textbf{Quartile-ranking} := \begin{cases} 1 & R^P \text{ is in the top 25\%} \\ 2 & R^P \text{ is in the top 50\%} \\ 3 & R^P \text{ is in the top 75\%} \\ 4 & R^P \text{ otherwise} \end{cases}$$

Floating bar chart



Problems with peer groups

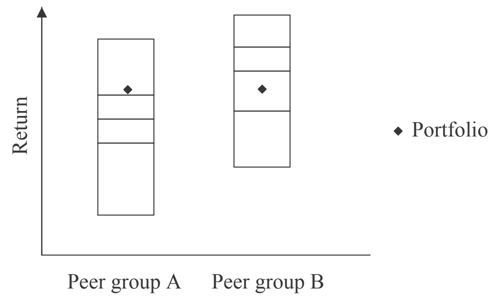
- **Survivorship bias**:
  Accounts terminated by underperformance are excluded from the database of the peer group provider.

  Longer measurement period $\Rightarrow$ data more biased.

  Marathon analogy (R. Surz, 1996):
  1000 runners,
  100 actually finish,
  is the 100th the last? Or in the top 10%?

Problems with peer groups

- **Classification bias**:
  When are mandates to be considered similar?
- **Composition bias**:
  Database provides too little observations:



These “biasses” cannot both be eleminated.

Contents

- Weaknesses of benchmarks and peer groups
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Portfolio Opportunity Distributions

- An alternative for benchmarks and peer groups:
- Eliminates all flaws inherent in the classical methods.
- Basic idea: compare the realized return with all returns that *could have been* realized.

Step 1

Specificy a **measurement period** and the manager's **mandate**:

Examples of mandate rules:

- Universe of investment securities.
- Bounds on exposure weights.
- Bounds on the number of different securities.
- Bounds on the weight in a certain class (region, country, sector,…)
- Bounds on ex-ante risk measures (tracking error).

Step 2

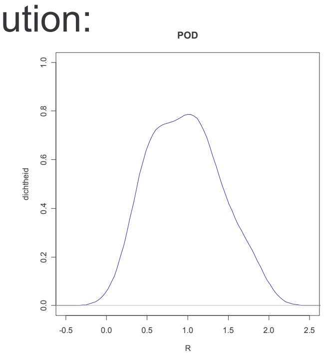
Generate a huge number of **random portfolios** which satisfy the manager's mandate.

Wall Street Journal, 1988:

Investors take the challenge to beat an imaginary gorilla (**market monkey**), which invests according to a random strategy.

Step 3

Compute the returns $R_1, \ldots, R_n$ of the sample portfolios:
This gives an estimation of a probability distribution:



Monte Carlo Simulation

Portfolio Opportunity Distrubution (POD)

Step 4

Apply Statistics:

R^P : realized return
$R_1, \ldots, R_n$: POD estimation
R : exact POD (stochastic variable)

POD ranking $\theta := \mathbb{P}(R > R^P)$

"probability that the monkey beats the investor"

POD ranking

Define: $$Y_i := \begin{cases} 1, & R_i > R^P; \\ 0, & R_i \le R^P. \end{cases}$$

Estimate: $$\hat{\theta}_n := \overline{Y}_n = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

Confidence interval:

$$\theta = \overline{Y}_n \pm \sqrt{\frac{\overline{Y}_n(1-\overline{Y}_n)}{n-1}}\Phi^{-1}(1-\alpha/2)$$

Did you beat the monkey?

Statistical test:

$$H_0: \quad \text{median}(R) \geq R^P$$
$$H_1: \quad \text{median}(R) < R^P$$

- R_i normally distributed: t-test:

$$T_n := \sqrt{n}\,\frac{\overline{R}_n - R^P}{\sigma_{R,n}}$$

- If not: sign test:

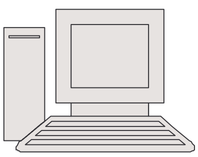
$$T_n := \#\{i : R_i < R^P\}$$

Reject the zero hypothesis at great values of T_n .

Summary

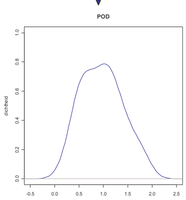
Mandate

Market data
(e.g. returns of securities)



MCS

R^P



Statistics → Performance

POD

Advantages w.r.t. benchmarks

- Per period a whole probability distribution instead of 1 numeric observation.
- All mandate rules are reflected, not only the investment universe.

Idea: POD mean is an idealized custom benchmark. Therefore benchmark analyses are also applicable for PODs.

Advantages w.r.t. peer groups

- No classification bias.
- No composition bias.
- No survivorship bias.

POD universum can be considered a “perfect” peer group, **but:** consists of random portfolios, not realistic portfolios.

Critical notes

- Comparison with a “monkey”, not with real competiting managers.
- Mandate must be well defined.
- No transactions (at the moment).
- Possibly a long run time is required by the computer simulation.

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Technical part

How can we design a procedure for generation of random portfolios?

- Model
- 3 algorithms

Model

$N :=$ # objects in our investment universe

Allocation: # segments
Selection: # securities

$W := (W^1, \ldots, W^N)$

vector of random weights, thus

$W^j \in [0\%, 100\%]$ and $\sum_{j=1}^{N} W^j = 100\%$

Model

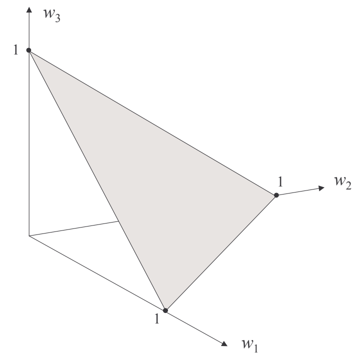
$R^j :=$ return of object j

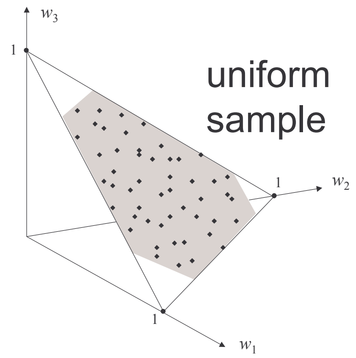
Allocation: benchmark return
Selection: security return

$R(W) = \sum_{j=1}^{N} W^j R^j$ return of the random portfolio

Model

Unit simplex ($N = 3$)



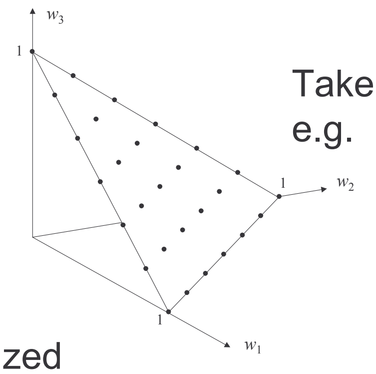


complete simplex

feasible set corresponding to the manager's mandate

Algorithm 1

No random samples, but an approximation of the exact POD:



Take discretization step d
e.g. $d = 0.01\%$

discretized
unit simplex

Algorithm 1

1. Loop over all vectors w for which the weights are a multiple of d .
2. **Acceptation/rejection**:
   determine if w satisfies all mandate rules.
   - Yes: Compute the return of w and add this to the data set.
   - No: Go to the next w.
3. The empirical distrubution, determined by the returns, approximates the exact POD.

Algorithm 1

Disadvantage: Complexity!
If $k := 100\% / d$ then the number of vectors to be looped over is

$$\binom{N+k-1}{k} = \frac{(N+k-1)!}{(N-1)!k!}$$

(exponential in N)

Because of this, algorithm 1 is only useful if $N \leq 5$.

Algorithm 2

1. Generate a uniform vector W from the N-dimensional unit simplex.
2. **Acceptation/rejection**:
   determine if W satisfies all mandate rules.
   Yes: Compute the return of W and add it to data set.
   No: Go to 1.
3. Data set big enough? Stop.
   Otherwise: Go to 1.

Algorithm 2

Procedure for uniform samples W from the N-simplex:

1. Generate N draws $E^1,\dots,E^N$ from the Exponential(1) distribution.
2. Compute the sum $$S := \sum_{j=1}^{N} E^j$$
3. $W := (W^1,\dots,W^N)$ where $W^j := E^j / S$

(**Theorem**: W is uniformly distrubuted on the N-simplex.)

Algorithm 2

Advantage: computation time is linear in the dimension N.

Disadvantage: not useful when acceptation/ rejectie procedure rejects most of the draws. Precize: if the rejection probability $\geq 99.999\%$ then it takes minutes to generate 1 vector.

Example: $N = 100$, weights max 2.6%.

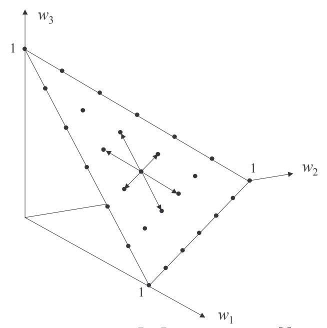
Algorithm 3

Markov Chain Monte Carlo (**MCMC**):

Idea: Simulate a stochastic process (Markov chain) on the feasible set in such a way that the limit distrubition is the uniform distribution.

Algorithm 3

State space: set of feasible points in the discretized unit simplex.



Take discretization step d
e.g. $d = 0.01\%$

State transitions: all possible transitions to a nearest neighbour (1 weight $+d$, 1 weight $-d$), unless not feasible.

Algorithm 3

How to garantee that the limit distrubution is uniform?

$(M_n)_n$: Nearest neighbour chain
Limit distribution: ???

$\downarrow$

$(X_n)_n$: Derivative chain
Limit distribution: uniform!

Algorithm 3

Hastings-Metropolis algorithm:

- Choose a begin state $X_0 = M_0$
- Given a state X_n:
  Generate a candidate state K according to the chain $(M_n)_n$: $\mathbb{P}(K = k) = m_{X_n,k}$
  With probability $\min\left(\frac{m_{K,X_n}}{m_{X_n,K}},1\right)$ accept K : $X_{n+1} := K$
  Otherwise, do nothing: $X_{n+1} := X_n$

(**Theorem**: limit distribution of $(X_n)_n$ is uniform.)

Algorithm 3

Advantage: generation directly from feasible set. Hence it works also for small feasible sets.

Disadvantage: computation time: # iterations needed per sample can be huge.
Example: $N \approx 100$, 16 hours for 100 draws.
Solution: split the universe of securities into groups, but this may influence the shape of the POD.

Conclusion: Use Algoritme 3 only if there is no alternative.

Which algorithm to use?

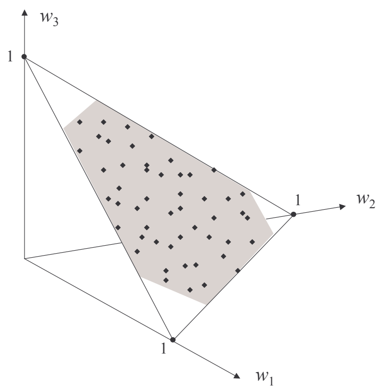
- $N \leq 5$: Alg 1 or 2.
- $N > 5$, big feasible set: Alg 2.
- $N > 5$, small feasible set: Alg 3.

Open problems

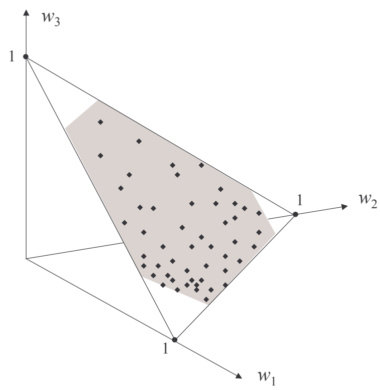
- Estimation of ex-ante tracking error.
- Simulation of transactions.
- Probability distributions.

Probability distributions

Which distribution on the feasible set?



Uniform distribution



Market distribution (capitalization weighted)

Probability distributions

Definition problem of market distribution. We require:

- Density is greater if the market capitalization

  $$\sum_{j=1}^{N} cap^j w^j$$

  is greater.
  Example: lineair density

  $$f_W(w) \sim \sum_{j=1}^{N} cap^j w^j$$

- Mean = benchmark:

  $$\mathbb{E}W \sim (cap^1, \ldots, cap^N)$$

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Examples

Illustration of the POD technique with 2 example investment mandates:

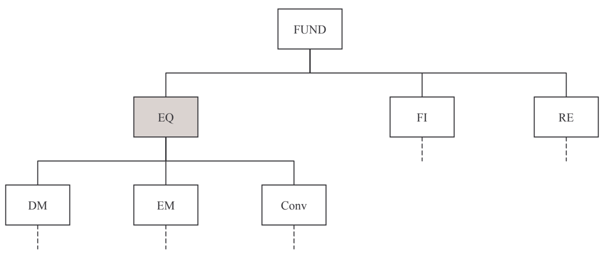
- Allocation to 3 equity segments.
- Selection out of 314 Japanese stocks.

Example 1

Equity segment of an asset management firm.

Allocation to 3 underlying segments:

- Developed Market (DM)
- Emerging Market (EM)
- Convertible bonds (Conv)



Example 1

Mandate rules:

- Ex-ante tracking error smaller or equal 5.5%.
- Exposure weights within these intervals

  DM: [77.8%, 91.6%];

  EM: [5.5%, 13.9%];

  Conv: [2.8%, 8.4%].

Measurement period: 2005-2006

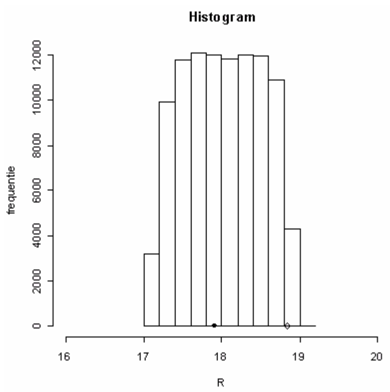
Realized returns (ann.):

- PF: 18.84%
- BM: 17.90%

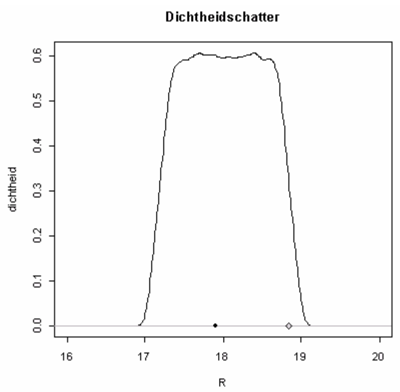
- DM: 16.40%
- EM: 35.11%
- Conv: 10.29%

Example 1

Portfolio Opportunity Distribution



histogram



density estimate

Example 1

- POD ranking:

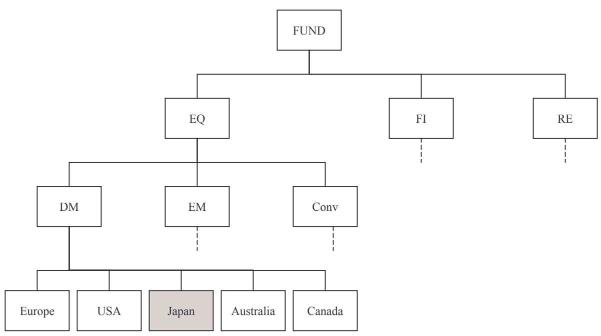
$\hat{\theta}_n = 0.0291$, with 95% conf.intv. $\theta \in [0.0281, 0.0302]$

- Sign test:

$p = 0$

Example 2

Selection mandate Japanese stocks.
Approximation: 314 MSCI Japan stocks.



Example 2

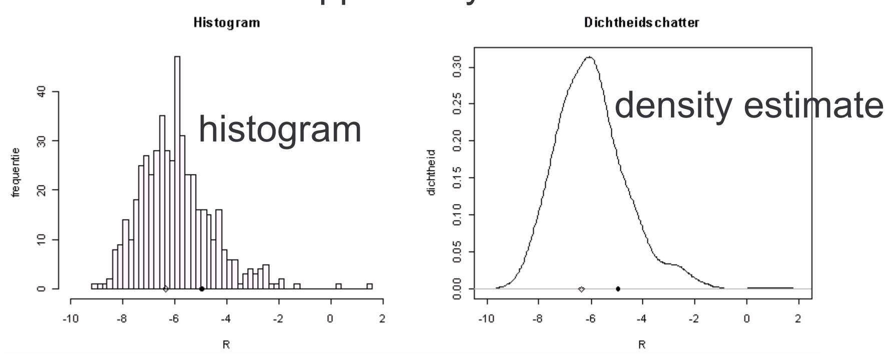
Mandate rules:
- Ex-ante tracking error smaller or equal 1.7%.
- Exposure weights within the interval [0%, 5%].
- Number of positive weights between 50 and 80.

Measurement period: 2006

Realized returns:
- PF: -6.359%
- BM: -4.959%

Example 2

Portfolio Opportunity Distribution



POD ranking: $\hat{\theta}_n = 0.584$, with 95% conf.intv.

$\theta \in [0.541, 0.627]$

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Conclusions

- PODs eliminate the flaws of benchmarks
  and peer groups.
  Reason: all mandate ruls are part of the input,
  and simulation gives many data within a short time.
- Interpretation: chosen strategy is compared
  with all possible “passive” strategies.
- Simulation if transactions is sometimes necessary
  but this leads to a problem of choice.
  Additional research will be required when ORTEC
  wishes to use POD techniques for a new product.